Transition Radiation and Optical Bremsstrahlung from Electron-Irradiated Thin Films of Gold, Silver, and Copper†*

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The optical emission from electron-irradiated self-supported films (500–2000 Å thick ±10%) of gold, silver, and copper has been investigated experimentally. Normally incident 1-μA beams of monoenergetic electrons having energies of 297, 423, 632 keV, and 1.10 MeV were used. The absolute photon intensity emitted in the 2200–5800-Å spectral region was measured with a calibrated optical system. The results were analyzed with respect to wavelength of emitted radiation, angular distribution in the forward direction (15°, 30°, and 45° from normal), and degree of polarization (parallel and perpendicular to the plane defined by the incident electrons and the emergent photons). The photon intensities measured in the perpendicular polarized direction clearly exhibit characteristics of bremsstrahlung. The transition-radiation contribution agrees very well with that predicted by the generalized theory of Ritchie and Eldridge when the bremsstrahlung is assumed to be unpolarized. From this, one may infer that interference between transition radiation and bremsstrahlung in the parallel plane does not grossly affect the emitted intensity. This investigation has extended the study of optical emission by Arakawa and co-workers to higher energies, thicker foils, and additional angles of emission.

I. INTRODUCTION

High-energy electrons passing through matter interact with both the nuclei and the atomic electrons within the material. These electrical interactions result in particle scattering and emission of electromagnetic radiation. Through the years, experimental and theoretical studies have revealed a broad class of radiation-producing phenomena. The visible and ultraviolet emissions which are relevant to the conditions of this experimental investigation are transition radiation and bremsstrahlung.

Transition radiation was first investigated theoretically by Frank and Ginzburg[1] during a period in which Russian theoreticians made an intensive study of radiation-producing phenomena. Their theory showed that a uniformly moving charged particle crossing a plane interface between two media of different dielectric constants generates radiation. Later, this principle was combined with aspects of plasma radiation by Ritchie and Eldridge[2] in a more generalized theory of transition radiation. In recent years, experimental investigations[3–5] have firmly established the validity of transition-radiation theory in the 6- to 100-keV incident-electron energy region. In particular, Arakawa *et al.*[5] have found that the spectral distribution and the absolute photon yield in the parallel plane agree with transition-radiation theory if the accompanying bremsstrahlung is assumed to be unpolarized. Only one group has considered higher electron energies, and in this case only integrated intensities were measured.

Bremsstrahlung is generated when energetic electrons moving in the field of ion cores within a material undergo scattering. The resultant radiation is affected by both the refractive and absorptive properties of the dielectric medium. Whereas the transition radiation is polarized parallel to the plane of the incident electrons and the observed photons, bremsstrahlung is radiated with both perpendicular and parallel components of polarization.

This paper reports the results of measurements of photon intensities emitted in the optical region when gold, silver, and copper films were irradiated with high-energy electrons, and compares the results with the theory of transition radiation and the characteristics of optical bremsstrahlung.

II. EXPERIMENTAL ARRANGEMENT

The experimental arrangement employed in measuring the optical emission from the electron-irradiated metallic films is shown in Fig. 1. The thin films of gold, silver, and copper located within the irradiation chamber were bombarded with 297-keV to 1.10-MeV electrons from a Dynamitron electron accelerator. The minimum energy employed was limited by the voltage stability of the electron accelerator. The collimated monoenergetic 1-μA beams passed normally through the thin-foil target into a Faraday collector. The currents were measured with an Elcor millimicroammeter. The light generated at the foil was collected by a 50-cm Seya-Namioka grating spectrometer which was alternately positioned at one of the three viewing ports (15°, 30°, and 45° in

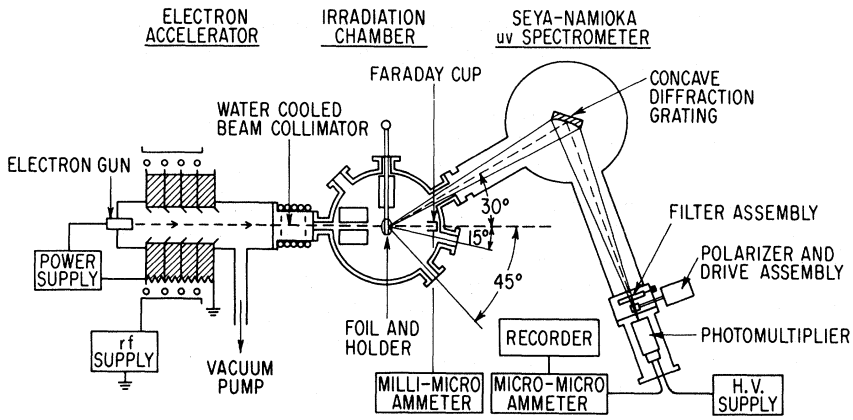

FIG. 1. Schematic diagram of the experimental arrangement showing accelerator, irradiation chamber, spectrometer, and detector.

the forward direction).

The metallic foils used in this investigation were prepared *in situ* thereby providing freshly evaporated surfaces for electron irradiation without exposure to the atmosphere. The different thicknesses of the metallic samples were obtained by using a self-supported film made as thin as possible (~350 Å) and of the same material as that to be deposited. Thereupon, additional material was deposited onto both sides of the foil producing successive thicknesses of approximately 500, 1000, 1500, and 2000 Å (± 10%) as desired. A reliable method was achieved which related film thickness to a known weight of each metal being evaporated. This was accomplished by relating evaporation parameters to resultant film thickness until a reproducible set of film thicknesses could be obtained. Film thicknesses were determined during these steps with a multiple-beam interferometer (Varian model No. 980-4000). The self-supported foils were made by evaporating the metal onto microscope slides which had been previously coated with a wetting agent. The thin film was then separated from the slide by floating it off onto a water surface. Finally, the film was picked up and mounted over the aperture of a demagnetized single-edged razor blade which was used as the substrate holder.

The mounted foil was placed at the center of the irradiation chamber. Since the foil was located at the focal point of the grating, the beam profile of 1.5-mm width and 5-mm height, defined by the collimator, acted as the entrance slit of the spectrometer. The resolution was limited by the width of the electron beam, 1.5 mm × 33 Å/mm or 50 Å for a 15 000-grooves/in. concave grating. The light emitted within a small solid angle was focused by the spectrometer through a Glan prism polarizer onto the cathode of an EMI 6256*S* quartz-window photomultiplier whose output was indicated on a Keithley dc picoammeter and recorded with a Brown chart recorder. The emitted intensity was recorded for two orientations (parallel and perpendicular) of the polarizer while the grating was scanned through the 2200–5800-Å spectral region. Absolute measurement of the radiation intensity in this spectral region required a calibrated optical system.

The absolute spectral response of the optical system was determined by measuring the solid angle subtended by the grating at the source, the grating efficiency in the first order, the transmission of the polarizer, the transmission of the quartz window, the transmission of the second-order filter, and the quantum efficiency and gain of the photomultiplier. The spectral responses of the optical components were determined over the 2200–5800-Å wavelength region. The absolute spectral response of the photomultiplier was determined for the 1900–5800-Å wavelength region with the aid of a calibrated thermopile (Epply Laboratory, serial No. 4499), a high-pressure mercury-lamp grating monochromator (Bausch and Lomb model No. 5), and a phosphor-coated (sodium salicylate) quartz plate. The calibrated thermopile was used for the 2536–5800-Å wavelength region. Below 2536 Å, the light intensity was too weak to be measured with the thermopile. As a consequence, an alternate method was used for the region below 2536 Å. The phosphor-coated quartz plate which exhibits a flat spectral response in the 900–3000-Å region was used in front of the photomultiplier. The results obtained in the 1900–3000-Å region were combined with the absolute response results derived from thermopile measurements above 2536 Å, extending the calibration to 1900 Å.

The Seya-Namioka spectrometer was calibrated

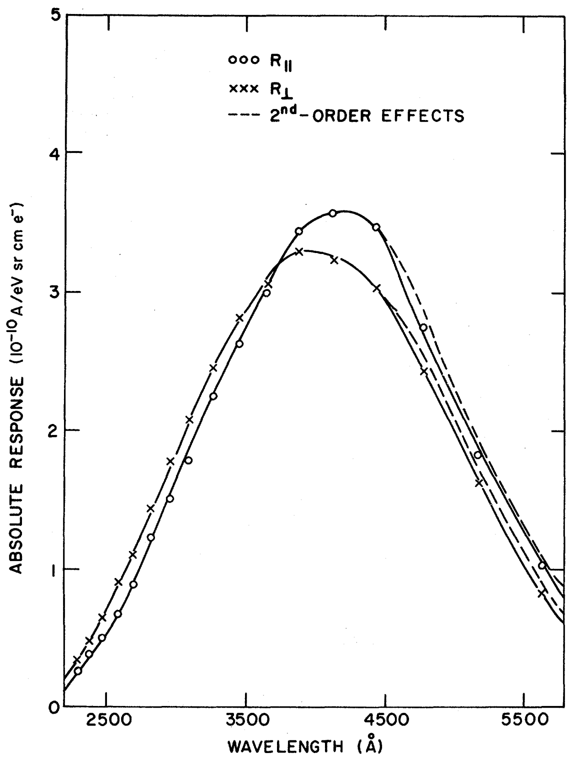

FIG. 2. Absolute spectral response of the optical system for light of both polarizations.

with respect to wavelength and absolute spectral response for both parallel and perpendicular polarized light with the aid of the mercury-lamp monochromator, Glan polarizer, and photomultiplier. In the process of measuring these parameters, the spectral transmissions of the polarizer, quartz window, and second-order filter were also measured. Figure 2 illustrates the absolute spectral response of the total optical system for light of both polarizations in the range 2200–5800 Å.

III. RESULTS

Radiation emitted within the 2200–5800-Å region from electron-irradiated foils of gold, silver, and copper was measured. The light levels were recorded for both polarizations; foil thicknesses of 500, 1000, 1500, and 2000 Å; electron energies corresponding to $\beta^2[=(v/c)^2]$ values of 0.60, 0.70, 0.80, and 0.90; and forward angles of emission of 15°, 30°, and 45°. By varying the parameters that occur in the theory of transition radiation we were able to determine the photon intensity dependence upon these parameters. Because of the large number of parameters varied in these experiments only typical results will be presented.

Since transition radiation is predicted to be polarized parallel to the plane of incidence, bremsstrahlung can be identified by the presence of a perpendicular polarized component in the experimental results. Bremsstrahlung, however, also contributes to the parallel polarized component observed. The relationship between the parallel and perpendicular polarized components has not been firmly established theoretically and experimentally. Gluckstern, Hull, and Breit[6] have treated the problem theoretically, and have derived an expression for low-energy bremsstrahlung generated by a high-energy electron scattering on an isolated screened nucleus of charge *Ze*. However, the effects of optical absorption and refraction could not be taken into account in their calculations. Shieh and Ritchie[7] recently have derived expressions for the simultaneous emission of transition radiation and bremsstrahlung generated by a high-energy electron scattered in a thin dielectric slab. The Gluckstern-Hull-Breit and the Shieh-Ritchie theories predict different degrees of bremsstrahlung polarization (both theories predict that the parallel component is weaker than the perpendicular component). The experimental results of Arakawa *et al.*[5] for thin gold foils and electron energies between 25 and 100 keV suggest that low-energy bremsstrahlung is unpolarized. These discrepancies concerning the degree of polarization of bremsstrahlung necessitate an analysis of both components of the measured photon intensity. Thus, we analyzed our experimental results in an attempt to resolve this conflict between the predicted and observed degrees of optical bremsstrahlung polarization. The hypothesis that the bremsstrahlung is unpolarized in the optical-wavelength region and under the conditions of this study led to the best agreement between the experimentally determined transition radiation and theory. Therefore, in presenting the experimental results and in comparing them with theory we have assumed the perpendicular polarized component of the radiation to be bremsstrahlung only and the parallel polarized component to be a combination of transition radiation and a bremsstrahlung component equal in intensity to the perpendicular bremsstrahlung intensity.

The theoretical photon intensity corresponding to transition radiation alone was computed from the theory of Ritchie and Eldridge using the optical constants of gold, silver, and copper obtained by Beaglehole[8] and by Fukutani and Sueoka,[9] by Taft and Philipp,[10] and by Ehrenreich and Philipp,[11] respectively. The Ritchie-Eldridge relation expresses the absolute photon intensity as a function of dielectric constant, angle of photon emission, foil thickness, wavelength of emitted light, and incident-electron energy. Calculations were made using a Control Data digital computer model No.

6600. The photon intensity distributions are given in units of electron volts per unit solid angle per unit wavelength per incident electron. In these units the Ritchie-Eldridge expression takes the form

$$I(\theta, a, \lambda, \beta) = \frac{2e^2\beta^2}{\pi\lambda^2} \cos^2\theta \sin^2\theta \left|\frac{\gamma}{\Delta}\right|^2 , \qquad (1)$$

where

$$\gamma = \left[\left(\frac{\beta\sigma + \epsilon}{1 - U^2\beta^2} - \frac{1}{1-\beta\sigma} \right) (U\epsilon + \sigma)\, e^{-it\sigma} + \left(\frac{\beta\sigma - \epsilon}{1 - U^2\beta^2} + \frac{1}{1+\beta\sigma} \right) (U\epsilon - \sigma)\, e^{it\sigma} - 2\sigma \left(\frac{\epsilon}{1 - U\beta} - \frac{1 + \beta\epsilon U}{1 - \beta^2\sigma^2} \right) e^{-it/\beta} \right] ,$$

$$\Delta = (U\epsilon - \sigma)^2\, e^{it\sigma} - (U\epsilon + \sigma)^2 e^{-it\sigma} ,$$

λ is the wavelength of emitted light, $\beta = v/c$, $\sigma = (\epsilon - 1 + U^2)^{1/2}$, $U = \cos\theta$, $t = 2\pi a/\lambda$, and ϵ is the complex dielectric constant. This result is valid for low-energy photons ($\hbar\omega \ll mc^2$) and for all velocities of the incident particle.

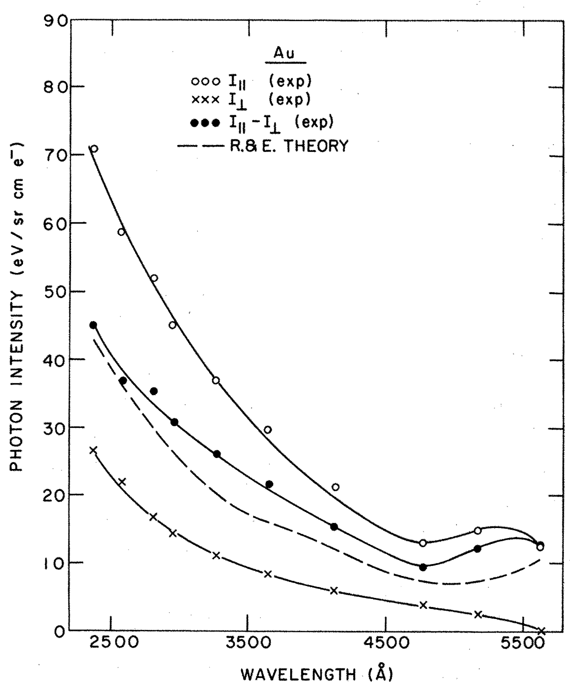

FIG. 3. Spectral distributions of radiation from a 500-Å gold foil; $\theta = 30°$, $\beta^2 = 0.6$. EXP (∥) is the experimental intensity in the parallel plane, EXP (⊥) is the experimental intensity in the perpendicular plane, and R and E theory is the transition-radiation intensity calculated from Ritchie and Eldridge theory.

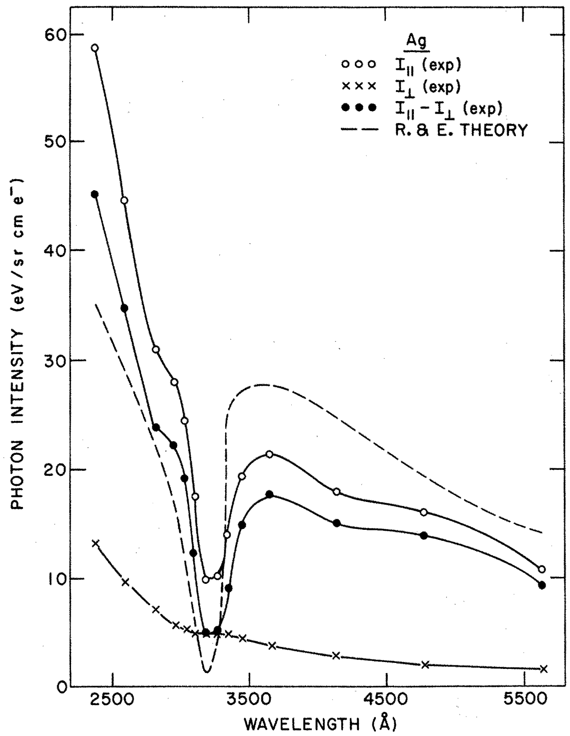

FIG. 4. Spectral distributions of radiation from a 500-Å silver foil; $\theta = 30°$, $\beta^2 = 0.6$. EXP (∥), EXP (⊥), and R and E theory have the same meaning as in Fig. 3.

Although the Shieh-Ritchie[7] theory is more complete than the Ritchie-Eldridge[2] theory, we have elected to compare our results with the latter calculations. The reasons for this decision will be explained in Sec. IV. We have confirmed that the term which corresponds to the emission of transition radiation in Shieh and Ritchie's paper is equivalent to that of Ritchie and Eldridge.

The spectral distributions of parallel and perpendicular polarized radiation between 2300 and 5630 Å emitted by gold, silver, and copper foils, approximately 500 Å thick are shown in Figs. 3–5, respectively. The photon intensities in these figures demonstrate the general property of an approximate $1/\lambda^2$ dependence. This property agrees very well with the expected wavelength dependence predicted by the transition-radiation theory of Ritchie and Eldridge, and the bremsstrahlung theory of Shieh and Ritchie, assuming the bremsstrahlung to be unpolarized. The distribution for silver shows a pronounced minimum at a wavelength of about 3200 Å. The minimum is related to the optical transparency of silver in this wavelength region. The sharp plasma peak observed at 3300 Å by other investigators using lower electron energies, is broadened out at these high energies.

The photon-intensity dependence upon foil thick-

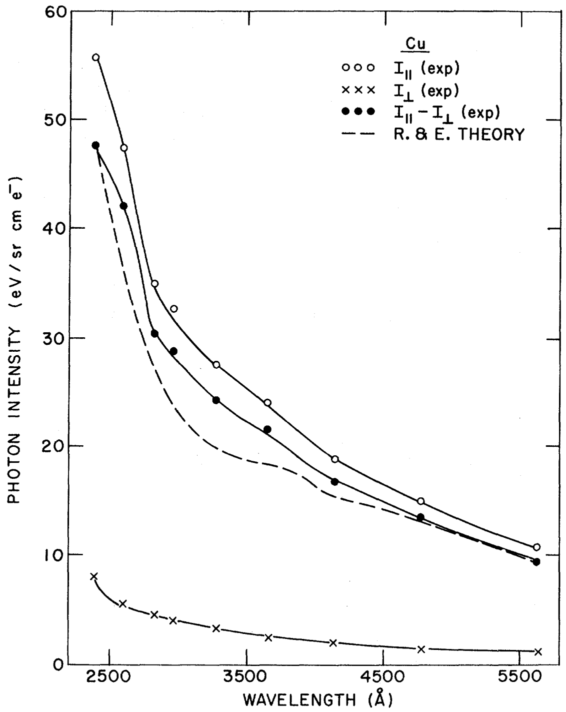

FIG. 5. Spectral distributions of radiation from a 500-Å copper foil; $\theta = 30°$, $\beta^2 = 0.6$. Notation has the same meaning as in Fig. 3.

ness is slightly different for the contributions from transition radiation and bremsstrahlung. The transition-radiation theory predicts an increase in intensity with thickness for very thin foils, the rate of increase of the intensity decreasing with thicker foils until it becomes essentially independent of thickness. The theory of bremsstrahlung predicts that the photon intensity varies linearly with the mean square scattering angle which in turn is almost linearly dependent on the thickness of the foil. The experimental results for copper are shown in Fig. 6 for a wavelength of 4130 Å, $\beta^2 = 0.80$, and an emission angle of 30°. As expected, the perpendicular polarized intensity shows a linear increase in intensity with foil thickness. The experimental results for transition radiation show good agreement with theory.

The angular dependence of the photon intensity is also different for the contributions from transition radiation and bremsstrahlung. According to the theory of Shieh and Ritchie, the angular distribution for bremsstrahlung is expected to be peaked more in the forward direction than that for transition radiation under the same experimental conditions. The results for silver, 2000 Å thick, at a radiation wavelength of 4130 Å, and an electron energy corresponding to $\beta^2 = 0.90$ are presented in Fig. 7. The perpendicular polarized intensity (bremsstrahlung) is seen to increase with decreasing angle of observation at a much faster rate than the corrected parallel component (transition radiation). The experimental and theoretical transition-radiation curves exhibit good agreement.

The theoretical dependence of the photon intensity on incident electron energy is considered to be quite different for transition radiation and bremsstrahlung. The theory of Ritchie and Eldridge predicts that the intensity varies approximately as β^2 at low electron energies ($\beta^2 \ll 1$). The bremsstrahlung theory presented by Gluckstern, Hull, and Breit predicts that the intensity varies as $1/\beta^2$ at low electron energies ($\beta^2 \ll 1$). The bremsstrahlung theory of Shieh and Ritchie provides an expression in which the intensity varies explicitly as β^2, with a large increase in intensity as $\beta \to 1$ and $\cos\theta \to 1$. Implicit in the same expression, however, is the dependence of the mean-square scattering angle upon the electron energy for a given foil thickness. An expression for the mean-square scattering angle as derived by Kalil *et al.*[12] pre-

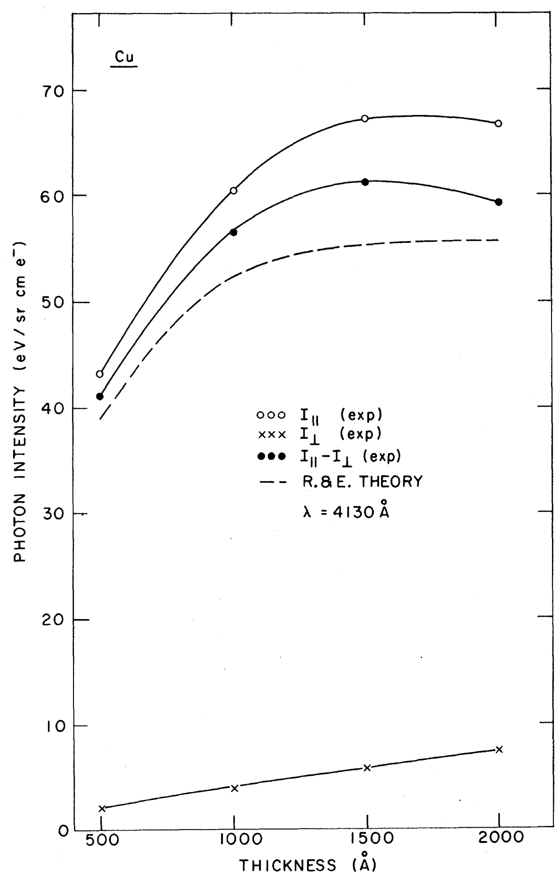

FIG. 6. Thickness dependence of radiation intensity emitted from copper foils; $\theta = 30°$, $\beta^2 = 0.8$, photon wavelength is 4130 Å. Other notation has the same meaning as in Fig. 3.

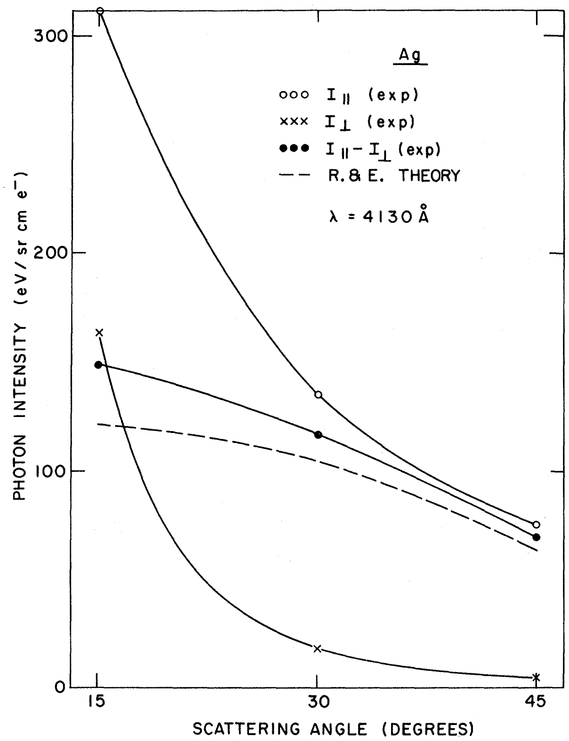

FIG. 7. Angular distribution of radiation from a 2000-Å silver foil; $\beta^2 = 0.9$, photon wavelength is 4130 Å. Other notation has the same meaning as in Fig. 3.

dicts a $1/\beta^4$ dependence. The results presented in Fig. 8 are for a gold foil 1000 Å thick, an emitted wavelength of 4130 Å, and an emission angle of 45°. The perpendicular polarized intensity is seen to increase only slightly with increasing energy (β^2). The corrected parallel polarized component agrees with the theoretically expected transition radiation.

The photon-intensity dependence on the material of the foil is considerably different for the contributions of transition radiation and bremsstrahlung. Whereas transition radiation involves electron interactions and is dependent essentially on the dielectric properties of the material, bremsstrahlung involves individual electron-screened nucleus interactions and therefore is very dependent on the atomic number Z of the material. The results presented in Figs. 9–11 show the dependence of photon intensity on the different materials studied. The results are presented for 2000-Å-thick gold, silver, and copper foils; an electron energy of $\beta^2 = 0.90$; and an emission angle of 15°. In Fig. 9, the perpendicular polarized spectral distributions are presented. The curves show the typical $1/\lambda^2$ distribution for each material. Comparison of the magnitudes of the distributions yields an approximate Z^2 ratio between gold and silver and a ratio between silver and copper which is closer to Z. Since the photon intensity for bremsstrahlung is also dependent on the optical properties of the material, the relative magnitudes of these distributions will not necessarily follow a Z^2 dependence (as expected from mean square scattering relations). The parallel polarized spectral distributions are presented in Fig. 10. The curves show evidence of a mixture of transition radiation and bremsstrahlung. In this figure, the spectral distributions for gold, silver, and copper are closer than in Fig. 9. The appearance of a minimum in the distribution for silver is also evident. In Fig. 11 the difference between the parallel and perpendicular polarized intensities, i.e., transition radiation, is presented. The magnitudes of the spectral distributions for gold, silver, and copper are comparable indicating the absence of any strong dependence on atomic number. Also, the minimum in the spectral distribution for silver appears more pronounced (affected by dielectric properties only).

IV. DISCUSSION AND CONCLUSIONS

The spectral yield in the type of experiment just described depends upon many parameters. Thus, one is tempted to reduce their number through various approximations. Since the early work of Frank and Ginzburg, many theoretical papers have

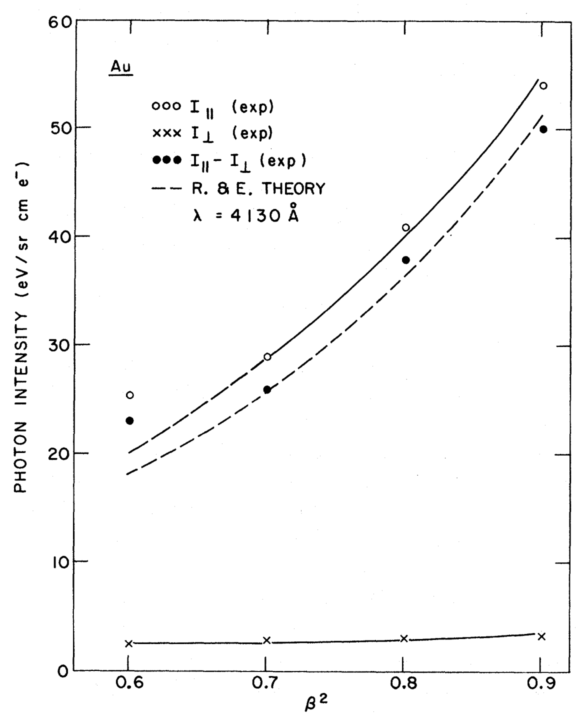

FIG. 8. Velocity dependence of emitted radiation from a 1000-Å gold foil; $\theta = 45°$, photon wavelength is 4130 Å. Other notation as in Fig. 3.

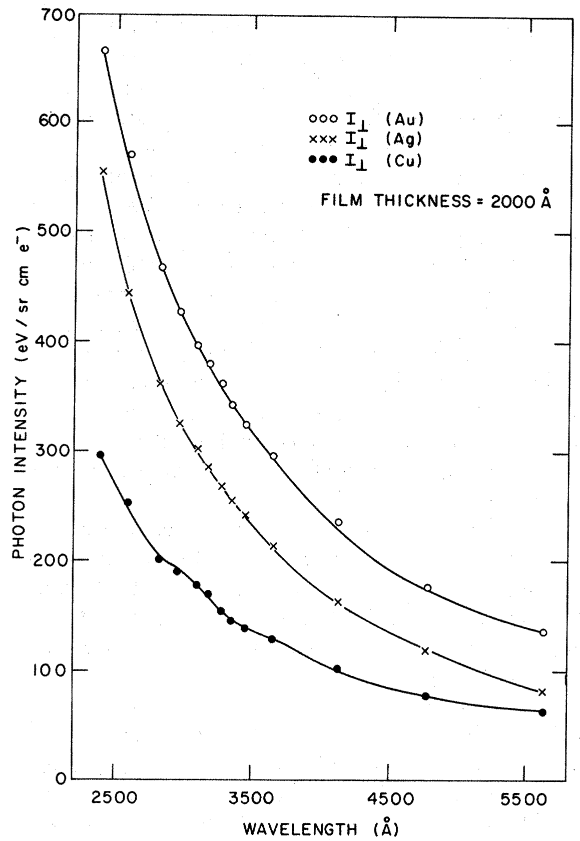

FIG. 9. Comparison of the spectral distributions of perpendicularly polarized (normal to the plane of incidence) radiation from 2000-Å foils of copper, silver, and gold; $\theta = 15°$, $\beta^2 = 0.9$.

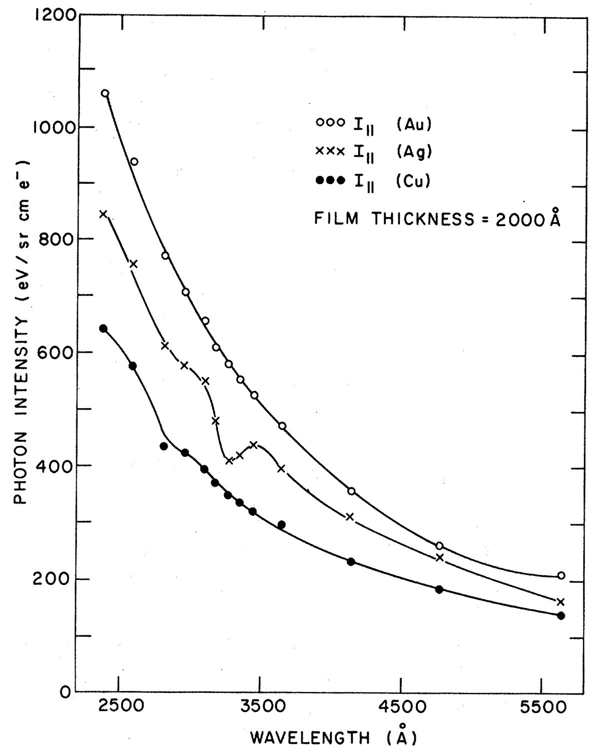

FIG. 10. Comparison of the spectral distributions of parallel polarized (in the plane of incidence) radiation from 2000-Å foils of copper, silver, and gold; $\theta = 15°$, $\beta^2 = 0.9$.

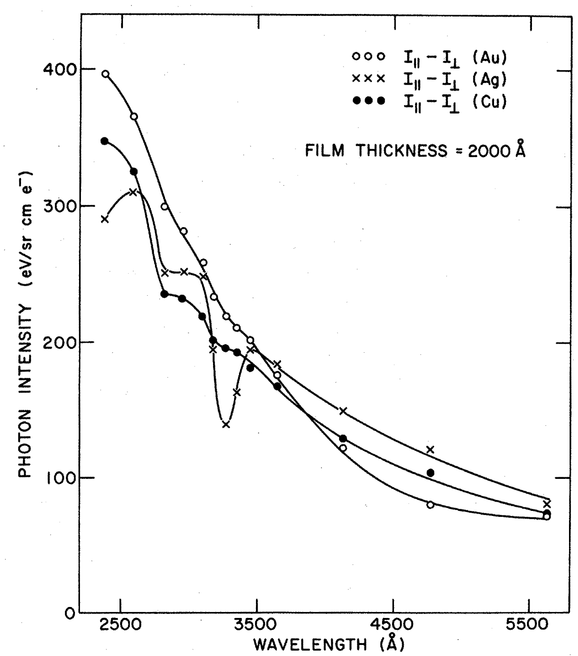

FIG. 11. Spectral distributions obtained by subtraction of those in Fig. 9 from the corresponding distributions in Fig. 10 for 2000-Å foils of copper, silver, and gold; $\theta = 15°$, $\beta^2 = 0.9$.

appeared in which reasonable approximations or limiting cases have been examined in detail. A recent survey by Pafomov[13] illustrates this proliferation of papers. It becomes necessary, then, to clearly define one's experimental conditions in order to select the appropriate theoretical treatment for comparison.

The salient feature of all of the theories is the interaction of the radiation fields induced by the charged particle with one another and with the field carried by the particle. The spectral distribution of each radiation field depends upon the dielectric response of the medium in which it is produced and upon the boundary conditions. That is, it depends upon polarization, absorption, refraction, and reflection. For the case of particles traversing a thin film, as in our experiments, the question of interference between light generated at the entrance and exit surfaces arises also. The importance of this effect depends upon the thickness of the film relative to the optical attenuation length λ_{opt}. The latter quantity represents the e-folding distance for absorption of light

TABLE I. Typical values of the optical attenuation length for copper, silver, and gold (in Å). (In the calculations, the angle between the direction of photon emission and the direction of the incident electron was taken as $\frac{1}{4}\pi$.)

λ(Å)	Cu	Ag	Au
2380	100	99	111
3100	145	162	140
4130	149	150	170
5630	164	124	175

of wavelength λ in a given medium, and is defined by the relation,

$$\lambda_{opt} \equiv \{2\,\mathrm{Re}[-(2\Pi i/\lambda)(\epsilon - 1 + \cos^2\theta)^{1/2}]\}^{-1} , \qquad (2)$$

where all of the quantities have the same meaning as in Eq. (1). In Table I are listed some typical values of λ_{opt} for copper, silver, and gold in the visible region of the spectrum when $\theta = \frac{1}{4}\pi$. Comparison of these values with our film thicknesses, viz., 500, 1000, 1500, and 2000 Å, indicates that our films are thick compared to λ_{opt} for visible light.

As stated, the experiments reported here were performed with a normally incident beam of electrons. However, the experimental observations were made on the beam-exit side of each target. Since the beam had to traverse the film before exiting, and since only light generated within a distance of the order of λ_{opt} from the exit surface could be observed, we examined further the assumption of normal incidence. This required an estimate of the multiple scattering in the films. We have made this estimate in two ways. First, using the Gaussian approximation to Molière's[14] theory as suggested by Bethe,[15] we have determined the mean-square scattering angle $\langle\theta^2\rangle$ for various combinations of film thickness and electron velocity. Second, for comparison we have performed a few calculations using an empirical expression given by Bothe[16] for the most probable angle of multiple scattering θ_λ. Bothe has shown that $\langle\theta^2\rangle = 2\theta_\lambda^2$ for a Gaussian distribution and for small scattering angles. The Bothe expression is

$$\theta_\lambda = \frac{8.0}{E}\,\frac{E+511}{E+1022}\,Z\left(\frac{\rho x}{A}\right)^{1/2} , \qquad (3)$$

where E is the kinetic energy of the electron, Z the atomic number, A the atomic weight of the target, ρ the density of the target, and x its thickness. If x is given in mg cm^{-2}, then Eq. (3) yields θ_λ in radians. Since multiple scattering increases with decreasing energy and is roughly proportional to thickness, the worst case in our experiments occurs when $\beta^2 = 0.6$ and the film thickness is 2000 Å. This combination is included in Table II where $\langle\theta^2\rangle$ is given on the basis of both Bethe and Bothe. The two estimates of $\langle\theta^2\rangle$ differ by no more than a factor of ~2 in all cases. Thus, we can assert that for our experiments $\langle\theta^2\rangle \ll 1$, and we may neglect electron backscattering. Moreover, because $\langle\theta^2\rangle$ is so small, we can regard the electron beam as normally incident on the exit face of the foils. From this conclusion one may infer that to a good approximation the perpendicular polarized component of emitted light has no transition-radiation admixture and may be treated as pure bremsstrahlung. This is only an approximation, of course, because if there were no scattering there would be no bremsstrahlung, and all observed light would be transition-radiation plane polarized in the parallel plane.

In view of the foregoing considerations, we concluded that the appropriate theory for comparison with our results would be that of Shieh and Ritchie although it assumes only single scattering. It treats the case of simultaneous transition radiation and bremsstrahlung emission from a foil of arbitrary thickness undergoing bombardment by normally incident charged particles of any energy. As expected, it predicts that only bremsstrahlung is emitted normal to the plane of incidence, whereas radiation in the plane of incidence consists of transition radiation, bremsstrahlung, and coherent interference between the two. Unfortunately, so far we have been unable to apply the Shieh-Ritchie expressions successfully to our results. We have programmed them for the CDC-6600 computer and have found that the interference terms predict intensities that are orders of magnitude larger than those we observe. The fact that these terms are

TABLE II. Calculated mean-square scattering angles for copper, silver, and gold films.

Element	β^2	t (mg cm^{-2})	$\langle\theta^2\rangle^a$	$\langle\theta^2\rangle^b$
Cu	0.6	4.5×10^{-2}	1.42×10^{-3}	3.2×10^{-3}
	0.6	1.8×10^{-1}	0.66×10^{-2}	1.3×10^{-2}
	0.7	9×10^{-2}	1.75×10^{-3}	
	0.8	1.35×10^{-1}	1.5×10^{-3}	
	0.9	1.8×10^{-1}	1.25×10^{-3}	1.47×10^{-3}
Ag	0.6	5.25×10^{-2}	2.15×10^{-3}	5.6×10^{-3}
	0.6	2.1×10^{-1}	1.08×10^{-2}	2.3×10^{-2}
	0.7	1.05×10^{-1}	2.7×10^{-3}	
	0.8	1.58×10^{-1}	2.2×10^{-3}	
	0.9	2.1×10^{-1}	1.08×10^{-3}	2.59×10^{-3}
Au	0.6	9.75×10^{-2}	0.71×10^{-2}	1.66×10^{-2}
	0.6	3.9×10^{-1}	3.38×10^{-2}	6.64×10^{-2}
	0.7	1.95×10^{-1}	0.94×10^{-2}	
	0.8	2.92×10^{-1}	0.72×10^{-2}	
	0.9	3.9×10^{-1}	0.37×10^{-2}	0.76×10^{-2}

[a]See Refs. 14 and 15.
[b]See Ref. 16.

also orders of magnitude larger than either the transition-radiation or bremsstrahlung terms suggests that there may be some invalid assumptions in the theory. The theoretical predictions for the perpendicular polarized yield are, conversely, much smaller than our experimental intensities. However, this was not unexpected, because we had first compared our observed bremsstrahlung yields with those predicted by the theory of Gluckstern, Hull, and Breit for isolated atoms.[6] Our experimental values exceeded the theoretical ones by one to two orders of magnitude although they exhibited approximately the expected dependence upon wavelength, thickness, and energy. As pointed out by Arakawa *et al.*,[5] since light transmitted through a dielectric medium is preferentially polarized in the plane of incidence, replacement of isolated atoms by a thin film is likely to reduce the intensity of the perpendicular component. This would only widen the discrepancy between theory and experiment.

Finally, then, we chose to follow the approach of previous experimental investigations in analyzing our data. We subtracted the perpendicular intensity from that in the plane of incidence, and compared the remainder with the transition-radiation theory of Ritchie and Eldridge.[2] We found that when the bremsstrahlung contribution was assumed to be unpolarized, the transition-radiation contribution to the parallel polarized component agreed well with that predicted by the theory. This observation is consistent with the earlier conclusions of Arakawa *et al.*,[5] also. It is perhaps worth noting at this point that the comparisons between theory and experiment did not involve any adjustable parameters. The optical constants were taken from the literature, and all other parameters were governed by the experimental conditions. The general agreement between the Ritchie-Eldridge theory and our data, therefore, suggests that optical bremsstrahlung intensity is much greater than that predicted by theory. Furthermore, we conclude that, except perhaps in silver, coherent interference effects are relatively small. In silver, they may be larger and negative. We infer this from the observation that for copper and gold the theoretical predictions are consistently smaller than the experimental values, whereas for silver the reverse is true over at least a portion of the spectrum. It has been suggested that experiments of the kind reported here can provide an independent means of measuring $\langle\theta^2\rangle$. However, until the question of bremsstrahlung intensities is resolved we do not anticipate their widespread use for this purpose.

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